Semi-Blind Joint Multi-CFO and Multi-Channel Estimation for GFDMA with Arbitrary Carrier Assignment

Yujie Liu*†, Xu Zhu*, Eng Gee Lim†, Yufei Jiang‡ and Yi Huang*
*Department of Electrical Engineering and Electronics, The University of Liverpool, Liverpool, UK
Email: {yujieliu, xuzhu, yufei.jiang, yi.huang}@liverpool.ac.uk
†Department of Electrical and Electronic Engineering, Xi'an Jiaotong-Liverpool University, Suzhou, P. R. China
Email: enggee.lim@xjtlu.edu.cn
‡School of Electronic and Information Engineering, Harbin Institute of Technology, Shenzhen, P. R. China

***Abstract*—We propose a low-complexity semi-blind joint multi-carrier frequency offset (CFO) and multi-channel estimation scheme for uplink generalized frequency division multiple access (GFDMA) systems. To the best of our knowledge, this is the first work to investigate the estimation of both CFOs and channels for a wide range of GFDMA systems, allowing arbitrary carrier assignment, modulation type and cyclic prefix length, and a wide range of the number of receive antennas. Thanks to the orthogonality between noise subspace and each signal subspace of U users, a complex U-CFO and U-channel estimation problem is decomposed into $2U$ one-dimensional problems, and solved in a semi-blind manner. Also, the multi-CFO compensation is performed at receiver rather than transmitter, avoiding spectral overhead due to feedback of multiple CFOs. Simulation results show that the proposed scheme significantly outperforms the existing methods in terms of bit error rate (BER) and root-mean-square-errors (RMSEs) of CFO and channel estimation, at much lower computational complexity than the existing methods.**

I. Introduction

Generalized frequency division multiple access (GFDMA) is a promising multiuser technique for future wireless communications [1], [2], which is a generalized form of orthogonal frequency division multiple access (OFDMA) and maintains most of the benefits of OFDMA while overcoming some of its challenges, e.g., large out-of-band (OOB) emission and high peak-to-average-power-ratio (PAPR). However, GFDMA suffers from multi-user interference (MUI), inter-carrier interference (ICI) and inter-symbol interference (ISI) caused by its nonorthogonal prototype filter. These kinds of interference become more severe in the presence of carrier frequency offsets (CFOs). A low-complexity zero-forcing receiver and a precoding technique to reduce PAPR were respectively proposed in [1] and [2] for uplink GFDMA. However, perfect channel estimation and frequency synchronization were assumed in [1] and [2]. The study on channel estimation and frequency synchronization for GFDMA is still an open area.

This work was supported in part by the AI University Research Centre (AI-URC) through XJTLU Key Programme Special Fund (KSF-P-02), and in part by the Natural Science Foundation of Guangdong Province under grant 2018A030313344.

A number of channel estimation [3]–[5] and frequency synchronization techniques [5]–[7] have been proposed for generalized frequency division multiplexing (GFDM) systems. In [3], [4], [6] and [7], only channel estimation or CFO estimation was considered, and their impacts on each other were ignored. In our previous work [5], a robust semi-blind scheme was proposed for joint estimation of CFO and channel for GFDM. Nevertheless, only a single user was considered in the aforementioned work, and thus these solutions cannot be applied to GFDMA systems with multiple users.

Frequency synchronization has been widely studied in OFDMA systems [8]–[12]. However, [8] and [9] were based on interleaved carrier assignment only. The blind CFO estimation approach in [10] was applicable for arbitrary carrier assignment, however, it works only under the conditions of constant modulus constellation or limited cyclic prefix (CP) length, and requires tremendous complexity for exhaustive search. In [11] and [12], the joint estimation of CFOs and channels for OFDMA with arbitrary carrier assignment was studied. However, the subspace based semi-blind approach proposed in [11] requires a large number of receive antennas as well as high complexity to search for CFOs, and the preamble-assisted scheme developed by Kalman and particle filtering in [12] reduces the spectral efficiency and has a much worse performance than the Cramér-Rao lower bound. In our previous work [13], a joint multi-time of arrival (TOA) and multi-CFO estimation scheme was developed for orthogonal frequency division multiplexing (OFDM) systems, assisted by carefully designed preambles. However, it requires high training overhead and ignores channel estimation. Furthermore, the aforementioned methods for OFDMA [8]–[12] or OFDM [13] cannot be applied to GFDMA, due to the nonorthogonal prototype filter of GFDMA.

In this paper, a low-complexity semi-blind joint multi-CFO and multi-channel estimation (SBJCCE) scheme is proposed for an uplink GFDMA system of U users with arbitrary carrier assignment. CFOs and channels are first separated into U groups by users using a subspace approach. For each user, the

CFO is extracted by minimizing the smallest eigenvalue whose corresponding eigenvector is utilized to estimate the channel in a blind manner. Finally, the channel ambiguity is eliminated by only a few pilots. Our contributions are as follows.

- To the best of our knowledge, this is the first work to investigate joint multi-CFO and multi-channel estimation for GFDMA systems. While the studies for GFDMA in [1] and [2] assumed perfect estimation of both CFOs and channels; the methods proposed in [3], [4], [6] and [7] and by us in [5] are for GFDM systems, not applicable to GFDMA with multiple users; the approaches developed for OFDMA [8]–[12] and for OFDM in our previous work [13] also cannot be applied to GFDMA due to the nonorthogonal filter of GFDMA. The proposed SBJCCE scheme for GFDMA significantly outperforms the existing methods for OFDMA in [10] and [11] in terms of bit error rate (BER) and root-mean-square-errors (RMSEs) of CFO estimation and channel estimation.
- The proposed SBJCCE scheme requires low complexity. Utilizing the orthogonality between noise subspace and each signal subspace, a complex $2U$-dimensional estimation problem is decomposed into $2U$ one-dimensional problems. The proposed CFO estimation algorithm reduces the complexity by up to hundreds of times over the methods in [10] and [11]. With separated CFOs, simple CFO compensation is also enabled for individual users at receiver side rather than transmitter side like [14].
- The proposed SBJCCE scheme requires low spectral overhead. The joint multi-CFO and multi-channel estimation scheme is performed semi-blindly, where only a few pilots are required for channel ambiguity elimination, leading to a much lower spectral overhead than the methods in [6] and [13]. The multi-CFO compensation at receiver side also avoids the signaling overhead to feedback CFOs to the respective transmitters in [14].
- The proposed SBJCCE scheme is applicable to a wide range of GFDMA systems. It supports arbitrary carrier assignment, modulation type and CP length, and also a wider range of the number of receive antennas than the method in [11]. While the existing methods can work only in OFDMA systems with an interleaved carrier assignment [8], [9], a constant modulus constellation [10] and a limited CP length [10].

The rest of this paper is organized as follows. Section II presents the system model. The proposed low-complexity SBJCCE scheme is described in Section III. Complexity analysis and simulation results are given in Sections IV and V, respectively. Section VI draws the conclusion.

Notations: Bold symbols represent vectors/matrices. Superscripts T and $*$ respectively denote the transpose and complex conjugate of a vector/matrix. diag$\{\mathbf{a}\}$ is a diagonal matrix with vector $\mathbf{a}$ on its diagonal. $\mathbf{0}_{M\times N}$ is an $M \times N$ zero matrix. $\|\cdot\|_{\mathrm{F}}^2$ is the Frobenius norm. $\otimes$ is the Kronecker product. $\mathbf{A}(a:b, c:d)$ is the submatrix of A with rows from a to b and columns from c to d. $\mathbf{A}'$ represents the estimate of $\mathbf{A}$.

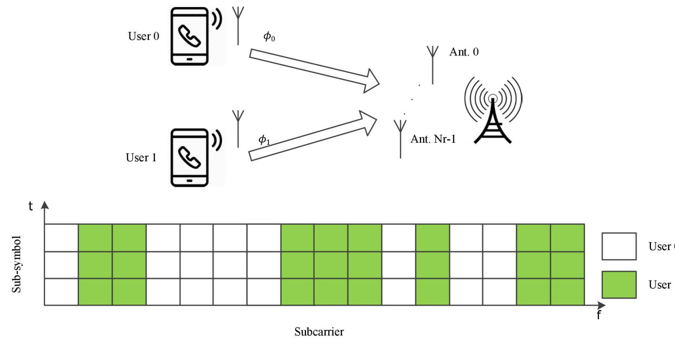

Fig. 1. An example of a multi-user SIMO GFDMA system with arbitrary carrier assignment in the presence of CFOs, with $U = 2$ users, $M = 3$ sub-symbols and $K = 16$ subcarriers.

II. System Model

We consider an uplink U-user single-input-multiple-output (SIMO) GFDMA system, where each user and the base station (BS) are equipped with a single transmit antenna and N_{r} receive antennas, respectively. Each GFDMA symbol is divided into M sub-symbols each with K subcarriers and we define $N = MK$. P arbitrary subcarriers are assigned to each user with $P = K/U$. The set of P subcarriers assigned to user u $(u = 0, 1, \cdots, U-1)$ is denoted as $\mathbf{K}_u$, where $\bigcup_{u=0}^{U-1} \mathbf{K}_u = \{0, 1, \cdots, K-1\}$ and $\mathbf{K}_u \bigcap \mathbf{K}_v = \varnothing$, $\forall u \neq v$. Fig. 1 illustrates an example of a SIMO GFDMA system with arbitrary carrier assignment in the presence of CFOs, with $U = 2$, $M = 3$ and $K = 16$.

Define $\mathbf{d}_{i,u} = [\mathbf{d}_{i,u,0}^T, \mathbf{d}_{i,u,1}^T, \cdots, \mathbf{d}_{i,u,M-1}^T]^T$, with $\mathbf{d}_{i,u,m} = [d_{i,u,m,0}, d_{i,u,m,1}, \cdots, d_{i,u,m,P-1}]^T$, where $d_{i,u,m,p}$ is the complex data of user u in the i-th symbol transmitted on the $\mathbf{K}_u(p)$-th subcarrier in the m-th sub-symbol. Each $d_{i,u,m,p}$ is transmitted with the corresponding pulse shape [1], [2]

$$g_{k,m}[n] = g[(n - mK) \bmod N] \cdot \exp(-j2\pi kn/K) \qquad (1)$$

where n $(n = 0, \cdots, N-1)$ is the sampling index. Note that each $g_{k,m}[n]$ is a time and frequency shifted version of a prototype filter $g[n]$. Denote $\mathbf{x}_{i,u} = [x_{i,u}[0], x_{i,u}[1], \cdots, x_{i,u}[N-1]]^T$ as the transmit data vector of u-th user in the i-th symbol, which can be expressed as

$$\mathbf{x}_{i,u} = \mathbf{A}(\mathbf{I}_M \otimes \mathbf{\Gamma}_u)\mathbf{d}_{i,u} \qquad (2)$$

where $\mathbf{A} = [\mathbf{g}_{0,0}, \cdots, \mathbf{g}_{K-1,0}, \cdots, \mathbf{g}_{0,M-1}, \cdots, \mathbf{g}_{K-1,M-1}]$ is an $N \times N$ pulse shaping filter matrix with $\mathbf{g}_{k,m} = [g_{k,m}[0], \cdots, g_{k,m}[N-1]]^T$, and $\mathbf{\Gamma}_u$ is the subcarrier assignment matrix of size $K \times P$ whose p-th column vector corresponds to the $\mathbf{K}_u(p)$-th column of the identity matrix $\mathbf{I}_N$. A single CP with length L_{cp} is pre-pended to the symbol $\mathbf{x}_{i,u}$, obtaining $\mathbf{s}_{i,u}$ of size $G = N + L_{\mathrm{cp}}$, which is given by $\mathbf{s}_{i,u} = \mathbf{\Psi}_u \mathbf{d}_{i,u}$ with $\mathbf{\Psi}_u = \mathbf{A}_{\mathrm{cp}}(\mathbf{I}_M \otimes \mathbf{\Gamma}_{\boldsymbol{u}})$ and $\mathbf{A}_{\mathrm{cp}} = [\mathbf{A}^T(N - L_{\mathrm{cp}} : N-1, 1 : N), \mathbf{A}^T]^T$.

The channel is assumed to exhibit quasi-static block fading and the channel impulse response (CIR) remains constant within a frame with N_{s} GFDMA symbols. Denote $\mathbf{h}_u^{n_{\mathrm{r}}} = [h_u^{n_{\mathrm{r}}}[0], h_u^{n_{\mathrm{r}}}[1], \cdots, h_u^{n_{\mathrm{r}}}[L-1]]^T$ as the CIR vector for the

n_r-th receive antenna of user u, with L being the length of CIR. ϕ_u is defined as the CFO between the u-th user and the BS. By incorporating the CFO into the transmitted signal and channel of each user [5], we obtain the CFO-included channel and CFO-included transmitted signal of user u given by $\bar{h}_u^{n_\mathrm{r}}[l] = e^{j2\pi\phi_u l/K} h_u^{n_\mathrm{r}}[l]$ and $\bar{s}_{i,u}[g] = e^{j2\pi\phi_u g/K} s_{i,u}[g]$, respectively. Thus, the time-domain received signal in the i-th symbol at the n_r-th receive antenna can be written as

$$y_i^{n_\mathrm{r}}[g] = \sum_{u=0}^{U-1} \sum_{l=0}^{L-1} \bar{h}_u^{n_\mathrm{r}}[l] \bar{s}_{i,u}[g-l] + w_i^{n_r}[g] \tag{3}$$

where $w_i^{n_r}[g]$ $(g = 0, \cdots, G-1)$ is the noise term.

The first $(L-1)$ signal samples $y_i^{n_\mathrm{r}}[0]$ to $y_i^{n_\mathrm{r}}[L-2]$ suffering from ISI due to CP are discarded and not utilized for estimation [5]. Collecting the received signal samples $y_i^{n_\mathrm{r}}[L-1]$ to $y_i^{n_\mathrm{r}}[G-1]$ from N_r received antennas into a vector, we obtain $\mathbf{y}_i = [y_i^0[L-1], \cdots, y_i^{N_\mathrm{r}-1}[L-1], \cdots, y_i^0[G-1], \cdots, y_i^{N_\mathrm{r}-1}[G-1]]^T$, which can be expressed as

$$\mathbf{y}_i = \sum_{u=0}^{U-1} \underbrace{\bar{\mathbf{H}}_u \mathbf{E}(\phi_u) \boldsymbol{\Psi}_u}_{\mathbf{G}_u} \mathbf{d}_{i,u} + \mathbf{w}_i \tag{4}$$

where $\mathbf{E}(\phi_u) = \mathrm{diag}\{\mathbf{e}(\phi_u)\}$ is the CFO matrix of user u with $\mathbf{e}(\phi_u) = [1, e^{j2\pi\phi_u/K}, \cdots, e^{j2\pi\phi_u(G-1)/K}]$, $\bar{\mathbf{H}}_u$ is the channel circulant matrix expressed as

$$\bar{\mathbf{H}}_u = \begin{bmatrix} \bar{\mathbf{h}}_u(L-1) & \cdots & \bar{\mathbf{h}}_u(0) & \cdots & \cdots & \mathbf{0}_{N_\mathrm{r}\times 1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0}_{N_\mathrm{r}\times 1} & \cdots & \cdots & \bar{\mathbf{h}}_u(L-1) & \cdots & \bar{\mathbf{h}}_u(0) \end{bmatrix} \tag{5}$$

with $\bar{\mathbf{h}}_u(l) = [\bar{h}_u^0[l], \cdots, \bar{h}_u^{N_\mathrm{r}-1}[l]]^T$, and $\mathbf{w}_i = [w_i^0[L-1], \cdots, w_i^{N_\mathrm{r}-1}[L-1], \cdots, w_i^0[G-1], \cdots, w_i^{N_\mathrm{r}-1}[G-1]]^T$.

III. Low-Complexity SBJCCE Scheme

Thanks to the orthogonality between noise subspace and each signal subspace of U-users, CFOs and channels are first separated into U groups. Second, for each user, CFO is first extracted by minimizing the smallest eigenvalue to satisfy the orthogonality, while the channel is determined by the corresponding eigenvector but with a scaling ambiguity which is then eliminated easily by a few pilots only. Fig. 2 illustrates the proposed SBJCCE scheme.

A. Multi-CFO Separation and Multi-Channel Separation

To allow the joint separation of CFOs and channels by user, the eigenvector corresponding to the noise subspace should be obtained. By exploiting N_r receive antennas with $(G-L+1)N_\mathrm{r} > N$, the frame with N_s received symbols is used to compute the auto-correlation of the received signal, obtaining $\mathbf{R}_\mathrm{y} = \frac{1}{N_\mathrm{s}} \sum_{i=0}^{N_\mathrm{s}-1} \mathbf{y}_i \mathbf{y}_i^H$. By performing eigenvalue decomposition (EVD) on the auto-correlation matrix $\mathbf{R}_\mathrm{y}$, the noise eigenvectors can be determined, which correspond to Q $(Q = (G-L+1)N_\mathrm{r} - N)$ smallest eigenvalues of $\mathbf{R}_\mathrm{y}$. The q-th noise eigenvector is expressed as $\boldsymbol{\gamma}_q = [\boldsymbol{\gamma}_q^T(0), \boldsymbol{\gamma}_q^T(1), \cdots, \boldsymbol{\gamma}_q^T(G-L)]^T$ where $\boldsymbol{\gamma}_q(g)$ is a column vector of size N_r. Due to the orthogonality between the noise

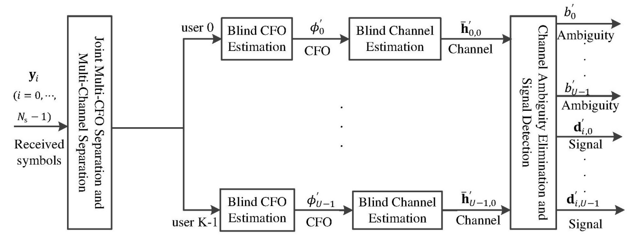

Fig. 2. Block diagram of the proposed SBJCCE scheme.

subspace and each signal subspace, the columns of $\mathbf{G}_u$ are orthogonal to each noise eigenvector, *i.e.*, $\boldsymbol{\gamma}_q^H \mathbf{G}_u = \mathbf{0}_{1\times MP}$. Since each user occupies exclusive subcarriers and has a different channel, $\mathbf{G}_u$ differs from each other. Define $\bar{\mathbf{h}}_u = [\bar{\mathbf{h}}_u^T(L-1), \bar{\mathbf{h}}_u^T(L-2), \cdots, \bar{\mathbf{h}}_u^T(0)]^T$ as the CFO-included channel vector of u-th user. The CFO and channel of u-th user are determined by

$$[\phi_u', \bar{\mathbf{h}}_u'] = \arg \min_{\phi_u, \bar{\mathbf{h}}_u} \sum_{q=0}^{Q-1} \| \boldsymbol{\gamma}_q^H \mathbf{G}_u \mathbf{G}_u^H \boldsymbol{\gamma}_q \|_\mathrm{F}^2 \tag{6}$$

Hence, by utilizing the orthogonality, the CFOs and channels can be separated by user. The complex multi-CFO compensation problem is thus divided into a number of simple single-CFO compensation problems, avoiding an additional signalling overhead to feedback CFOs to the transmitters in [14]. It is noteworthy that the proposed scheme has a looser requirement on the CP length and number of receive antennas. The CP length in the proposed scheme can be chosen as any value, while it in [10] is limited to $L_\mathrm{cp} < \frac{P-1}{2}$ with P being the number of subcarriers per user. The proposed SBJCCE scheme also requires a smaller number of receive antennas to meet $(G-L+1)N_\mathrm{r} > N$ than the approach in [11] whose required number of receive antennas is linearly proportional to the number of users and the channel length.

B. Blind CFO and Channel Estimation

After the separation of CFOs and channels by user, each C-FO is first extracted to satisfy the orthogonality by minimizing the smallest eigenvalue, whose corresponding eigenvector is exploited to determine the corresponding channel. To avoid an exhaustive search for CFO, we then propose an efficient CFO estimator by exploiting the properties of the well-designed cost function.

Define $\mathbf{R}_u(\tilde{\phi}_u) = \| \boldsymbol{\gamma}_q^H \mathbf{G}_u(\tilde{\phi}_u) \mathbf{G}_u^H(\tilde{\phi}_u) \boldsymbol{\gamma}_q \|_\mathrm{F}^2$, where $\mathbf{G}_u(\tilde{\phi}_u) = \bar{\mathbf{H}}_u \mathbf{E}(\tilde{\phi}_u) \boldsymbol{\Psi}_u$, with $\tilde{\phi}_u$ as the CFO trial value of u-th user. $\mathbf{R}_u(\tilde{\phi}_u)$ can be written as

$$\mathbf{R}_u(\tilde{\phi}_u) = \boldsymbol{\gamma}_q^H \bar{\mathbf{H}}_u \mathbf{E}(\tilde{\phi}_u) \boldsymbol{\Psi}_u \boldsymbol{\Psi}_u^H \mathbf{E}^H(\tilde{\phi}_u) \bar{\mathbf{H}}_u^H \boldsymbol{\gamma}_q \tag{7}$$

According to [15], $\bar{\mathbf{H}}_u$ as a Toeplitz matrix, has a property of $\boldsymbol{\gamma}_q^H \bar{\mathbf{H}}_u = \bar{\mathbf{h}}_u^T \boldsymbol{\Upsilon}_q$ where $\boldsymbol{\Upsilon}_q$ of size $N_\mathrm{r}L \times G$ is given by

$$\boldsymbol{\Upsilon}_q = \begin{bmatrix} \boldsymbol{\gamma}_q^*(0) & \cdots & \boldsymbol{\gamma}_q^*(G-L) & \cdots & \cdots & \mathbf{0}_{N_\mathrm{r}\times 1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0}_{N_\mathrm{r}\times 1} & \cdots & \cdots & \boldsymbol{\gamma}_q^*(0) & \cdots & \boldsymbol{\gamma}_q^*(G-L) \end{bmatrix} \tag{8}$$

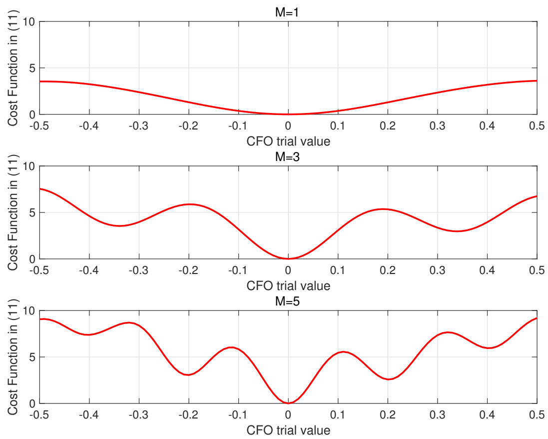

Fig. 3. Cost function $\beta_u(\tilde{\phi}_u)$ in (11) as a function of CFO trial value $\tilde{\phi}_u$, with $\phi_u = 0$, $M = 1$, 3 and 5.

Therefore, (7) can be rewritten as

$$\mathbf{R}_u(\tilde{\phi}_u) = \bar{\mathbf{h}}_u^T \underbrace{\boldsymbol{\Upsilon}_q \mathbf{E}(\tilde{\phi}_u) \boldsymbol{\Psi}_u}_{\mathbf{P}_{u,q}(\tilde{\phi}_u)} \underbrace{\boldsymbol{\Psi}_u^H \mathbf{E}^H(\tilde{\phi}_u) \boldsymbol{\Upsilon}_q^H}_{\mathbf{P}_{u,q}^H(\tilde{\phi}_u)} \bar{\mathbf{h}}_u^* \tag{9}$$

Define $\mathbf{P}_u(\tilde{\phi}_u) = [\mathbf{P}_{u,0}(\tilde{\phi}_u), \mathbf{P}_{u,1}(\tilde{\phi}_u), \cdots, \mathbf{P}_{u,Q-1}(\tilde{\phi}_u)]$. The CFO and channel of user u can be estimated by

$$[\phi'_u, \bar{\mathbf{h}}'_u] = \arg \min_{\phi_u, \bar{\mathbf{h}}_u} \| \bar{\mathbf{h}}_u^T \mathbf{P}_u(\tilde{\phi}_u) \mathbf{P}_u^H(\tilde{\phi}_u) \bar{\mathbf{h}}_u^* \|_{\mathrm{F}}^2 \tag{10}$$

Then, the CFO and channel of user u are estimated in the following steps:

Step 1: The auto-correlation matrix of $\mathbf{P}_u(\tilde{\phi}_u)$ is calculated by $\mathbf{R}_{\mathbf{P},u}(\tilde{\phi}_u) = \mathbf{P}_u(\tilde{\phi}_u)\mathbf{P}_u^H(\tilde{\phi}_u)$. If $\tilde{\phi}_u$ is the true CFO ($\tilde{\phi}_u = \phi_u$), the rank of $\mathbf{R}_{\mathbf{P},u}(\tilde{\phi}_u)$ is $(N_{\mathrm{r}}L - 1)$, otherwise the rank is $N_{\mathrm{r}}L$. Hence, the CFO of user u can be extracted by

$$\phi'_u = \min_{\tilde{\phi}_u \in [-0.5, 0.5)} \beta_u(\tilde{\phi}_u) \tag{11}$$

with $\beta_u(\tilde{\phi}_u)$ defined as the smallest eigenvalue of $\mathbf{R}_{\mathbf{P},u}(\tilde{\phi}_u)$.

Note that CFO estimation directly based on exhaustive search in (11) requires a large number of computations. Fig. 3 illustrates the cost function $\beta_u(\tilde{\phi}_u)$ as a function of CFO trial value, with $\phi_u = 0$, $M = 1$, 3 and 5. We can observe that the cost function has M minimums corresponding to the number of sub-symbols per GFDMA symbol, and the distance between any two neighbor minimums is approximately $1/M$. Hence, by utilizing these properties, the low-complexity CFO estimation can be achieved following the steps below:

Step 1.1: Utilize the golden section search and parabolic interpolation algorithms to search for the minimum within the range $[-0.5/M, 0.5/M)$, denoted as ϕ_u^0.

Step 1.2: The rest minimums can be determined by utilizing the relationship between any two neighbour minimums, denoted as $\phi_u^1, \cdots, \phi_u^{M-1}$.

Step 1.3: Put all the minimums back to the cost function $\beta_u(\tilde{\phi}_u)$ in (11), and select the one corresponding to the smallest value of $\beta_u(\tilde{\phi}_u)$, denoted as ϕ_u^{m}.

Step 1.4: As the distance between any two neighbour minimums is not accurately $1/M$ due to the noise, Step 1.3 might suffer an error floor. We then utilize the golden section search and parabolic interpolation algorithms to search for the minimum within the range $[\phi_u^{\mathrm{m}} - 0.5/M, \phi_u^{\mathrm{m}} + 0.5/M)$. We denote the final CFO estimate as ϕ'_u.

Step 2: After obtaining the CFO estimate ϕ'_u, the channel of user u is determined as the conjugate of the eigenvector corresponding to its smallest eigenvalue of $\mathbf{R}_{\mathbf{P},u}(\phi'_u)$, denoted as $\bar{\mathbf{h}}'_{u,0}$. It is worthy noticing that a complex scaling ambiguity b_u exists between the blind channel estimate $\bar{\mathbf{h}}'_{u,0}$ and the true channel $\bar{\mathbf{h}}_u$, which can be eliminated easily by a few pilots.

C. Channel Ambiguity Elimination and Signal Detection

With the blind estimates of CFO and channel ϕ'_u and $\bar{\mathbf{h}}'_{u,0}$, $\mathbf{G}'_u$ can be easily obtained according to (4). The received signal $\mathbf{y}_i$ can be expressed as

$$\mathbf{y}_i = \sum\nolimits_{u=0}^{U-1} \mathbf{G}'_u \mathbf{d}_{i,u} b_u + \mathbf{w}_i \tag{12}$$

Define $\mathbf{G}' = [\mathbf{G}'_0, \mathbf{G}'_1, \cdots, \mathbf{G}'_{U-1}]$. The received signal vector $\mathbf{y}_i$ is multiplied with the pseudoinverse of $\mathbf{G}'$, obtaining $\mathbf{r}_i$. Denote $\mathbf{r}_i = [\mathbf{r}_{i,0}^T, \mathbf{r}_{i,1}^T, \cdots, \mathbf{r}_{i,U-1}^T]^T$, and $\mathbf{r}_{i,u}$ is given by

$$\mathbf{r}_{i,u} = \mathbf{d}_{i,u} b_u + \hat{\mathbf{w}}_{i,u} \tag{13}$$

with $\hat{\mathbf{w}}_{i,u}$ being the noise vector. Assuming P_{pil} pilots in the first GFDMA symbol ($i = 0$) are utilized for ambiguity elimination, the scaling ambiguity of user u is estimated by

$$b'_u = \frac{1}{P_{\mathrm{pil}}} \sum\nolimits_{p=0}^{P_{\mathrm{pil}}-1} \frac{\mathbf{r}_{0,u}(p)}{\mathbf{d}_{0,u}(p)} \tag{14}$$

Hence, after ambiguity elimination, the channel is obtained as $\bar{\mathbf{h}}'_u = \bar{\mathbf{h}}'_{u,0} b'_u$. Then, the signal of user u is determined as $\mathbf{d}'_{i,u} = \mathbf{r}_{i,u}/b'_u$. Note that the proposed SBJCCE scheme is independent of the modulation type, while the method in [10] is working only with a constant modulus constellation.

IV. Complexity Analysis

Since the existing methods in OFDMA systems with arbitrary carrier assignment [10], [11] mainly focus on CFO estimation, we compare their complexities with that of the proposed SBJCCE scheme in CFO estimation only. Their symbolic computational complexities are demonstrated in Table I, in terms of the number of complex additions and multiplications. It is noteworthy that both Wang's [10] and Zhang's [11] schemes are based on an exhaustive search and their complexities depend on the search step size Δ. Meanwhile, Wang's scheme [10] estimates the CFOs of all users in an iterative manner, whose complexity is linearly proportional to the number of iterations N_{itr}. Regarding the proposed SBJCCE scheme, all CFOs are estimated separately and does not require an iterative and exhaustive search. We count the number of cost function evaluations when implementing the golden section search and parabolic interpolation algorithms, and denote it as a which is shown to be less than 20 from simulation. The numerical complexities of the proposed scheme and the existing methods [10], [11] are calculated, based on the simulation setup in Section V, except for M, where M is chosen as 1 for the proposed SBJCCE scheme. It is found that the proposed scheme is very computationally efficient, with a complexity reduction of around 170-fold and 52-fold over that of Wang's [10] and Zhang's [11] schemes.

TABLE I

Computational Complexity for CFO Estimation (N_r: Number of receive antennas, N_s: Number of symbols in a frame, K: Number of subcarriers per GFDMA/OFDMA symbol, M: Number of sub-symbols per GFDMA symbol, U: Number of users, L: Channel length, L_cp: CP length, $G = KM + L_\mathrm{cp}$, $Q = (G - L + 1)N_\mathrm{r} - MK$, a: Number of times to compute the cost function in (11), Δ: CFO search step size in [10] and [11] and N_itr: Number of iterations in [10]).

Item	CFO Estimation
SBJCCE scheme	$(2G^2N_\mathrm{r}L + 2GN_\mathrm{r}LMP + 2N_\mathrm{r}^2L^2MP)QUa$ $2(G-L+1)^2N_\mathrm{r}^2N_\mathrm{s} + (G-L+1)^3N_\mathrm{r}^3$
Wang's scheme - OFDMA [10]	$N_\mathrm{s}N_\mathrm{r}K\log_2 K + (6K^3U + 2K^2N_\mathrm{s} + (N+2PN + 2P^2 + 2P)N_\mathrm{s}U)$ UN_itr/Δ
Zhang's scheme - OFDMA [11]	$(2K^2N_\mathrm{s}N_\mathrm{r} + N_\mathrm{s}N_\mathrm{r}K\log_2 K + 2N_\mathrm{s}N_\mathrm{r}(K-K/U)K + N_\mathrm{r}^3 + 2N_\mathrm{r}^2(K-K/U)N_\mathrm{s})U/\Delta$

V. Simulation Results

Monte Carlo simulations have been carried out to demonstrate the performance of the proposed SBJCCE scheme for SIMO GFDMA systems with arbitrary carrier assignment. Up to 5000 realizations are performed each with a random carrier assignment. Since CFO and channel estimation for GFDMA systems are lacking in the literature, the methods for OFDMA systems in [10] and [11] are utilized as benchmarks. System parameters are specified as follows: each GFDMA symbol contains $M = 3$ sub-symbols, except for Fig. 7; each sub-symbol includes $K = 16$ subcarriers; the number of users is $U = 2$; the receiver is equipped with $N_\mathrm{r} = 3$ antennas; the CP length is $L_\mathrm{cp} = 4$; the channel model follows an exponential profile with length $L = 2$; the number of symbols utilized for blind channel estimation is $N_\mathrm{s} = 128$; the number of pilots for channel ambiguity elimination is $P_\mathrm{pil} = 10$, except for Fig. 5; the CFOs are randomly generated within $[-0.4, 0.4)$ [11]; root raised cosine prototype filter with roll-off coefficient $\alpha = 0.5$ is utilized for GFDMA; quadrature phase shift keying (QPSK) modulation scheme is utilized. Based on [10] and [11], the search step size Δ for Wang's and Zhang's schemes is selected as 0.001, while the number of iterations N_itr for Wang's scheme is chosen as 5. The root mean square errors (RMSEs) of CFO and channel estimation are defined as $\mathrm{RMSE}_\mathrm{CFO} = \sqrt{\mathbb{E}\{\frac{1}{U}\sum_{u=0}^{U-1}(\phi'_u - \phi_u)^2\}}$ and $\mathrm{RMSE}_\mathrm{channel} = \sqrt{\mathbb{E}\{\frac{1}{UN_\mathrm{r}L}\sum_{u=0}^{U-1} \| \bar{\mathbf{h}}'_u - \bar{\mathbf{h}}_u \|_\mathrm{F}^2\}}$.

Fig. 4 demonstrates the RMSE performance of CFO estimation of the proposed SBJCCE scheme, in comparison to Wang's scheme [10] and Zhang's scheme [11] for OFDMA.

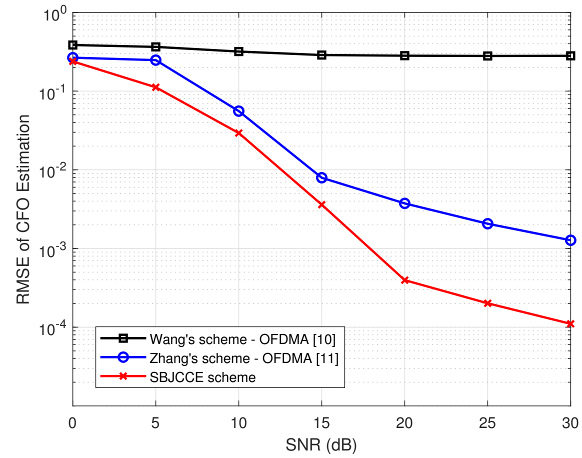

Fig. 4. RMSE of CFO estimation of the proposed SBJCCE scheme.

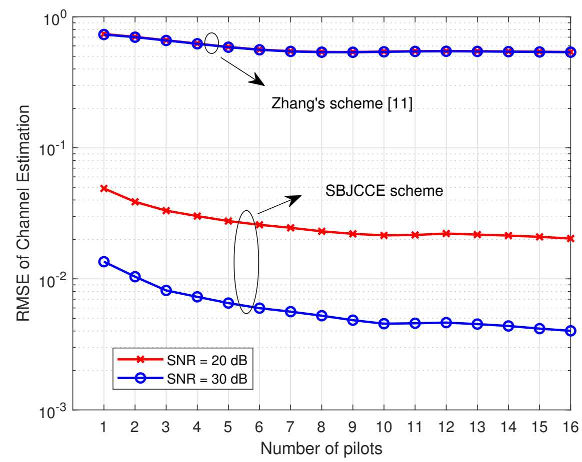

Fig. 5. Impact of the number of pilots on RMSE of channel estimation of the proposed SBJCCE scheme.

The proposed SBJCCE scheme provides a much better performance than the existing methods [10], [11], especially at high signal-to-noise-ratios (SNRs). For instance, at the RMSE of CFO estimation of 10^{-3}, the proposed scheme achieves an SNR gain of approximately 12.5 dB over Zhang's scheme [11]. Wang's scheme [10] suffers from an error floor. This is because it is working only with a very short CP length.

Fig. 5 shows the impact of the number of pilots on RMSE of channel estimation of the proposed SBJCCE scheme and Zhang's scheme [11], at SNR $=$ 20 dB and SNR $=$ 30 dB, respectively. It is easily seen that Zhang's scheme has a very poor performance in channel estimation, regardless of the number of pilots. This poor performance results from the shortage of antennas at the receiver with $N_\mathrm{r} = 3$, while the proposed scheme with $N_\mathrm{r} = 3$ can achieve a much better performance. The proposed scheme achieves a convergence with 10 pilots.

Fig. 6 provides the BER performance of the proposed

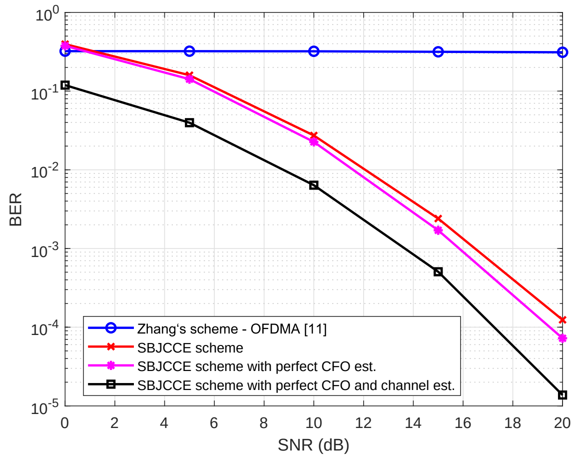

Fig. 6. BER performance of the proposed SBJCCE scheme (est.: estimation).

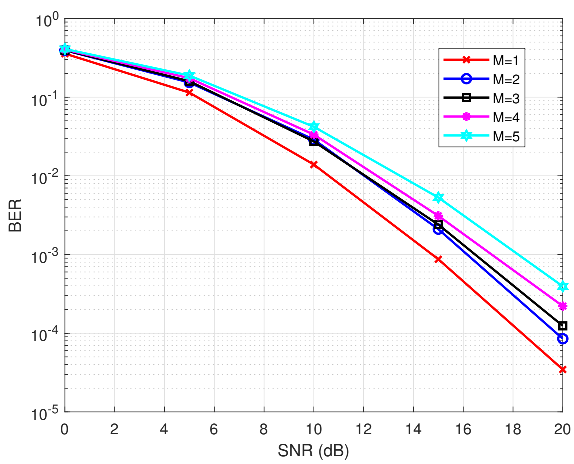

Fig. 7. Impact of the number of sub-symbols per GFDMA symbol on BER performance of the proposed SBJCCE scheme.

SBJCCE scheme and Zhang's scheme [11]. The proposed scheme with perfect estimation of CFO and perfect estimation of both CFO and channel are utilized as benchmarks. It is easily seen that the proposed scheme shows a slightly worse performance than its counter part with perfect CFO estimation. At the BER performance of approximately 10^{-4}, the proposed scheme with perfect CFO estimation achieves an SNR gain lower than 1 dB over the proposed scheme. Zhang's scheme [11] still demonstrates an error floor because it is dependent on a larger number of receive antennas.

Fig. 7 studies the impact of the number of sub-symbols per GFDMA block on BER performance of the proposed SBJCCE scheme. We can see the proposed scheme performs slightly worse as M increases. The performance degradation mainly results from channel estimation. The size of GFDMA symbol increases with M, which then requires more received symbols to obtain the second-order statistics of the received signal for blind channel estimation.

VI. Conclusion

A low-complexity semi-blind joint multi-CFO and multi-channel estimation scheme has been proposed for a wide range of GFDMA systems. The proposed scheme has high spectral efficiency, requiring only a small number of pilots to eliminate channel ambiguities, and no signaling feedback due to multi-CFO compensation performed at receiver side. It significantly outperforms Wang's [10] and Zhang's [11] methods in OFDMA systems in terms of BER and RMSEs of CFO estimation and channel estimation, without suffering any error floors in [10] and [11] due to the long CP and shortage of receive antennas, respectively. The proposed scheme demonstrates a close BER performance to its counterpart with perfect CFO estimation at all SNRs. It also has a complexity reduction of 170 and 52 times over Wang's [10] and Zhang's approaches [11] in CFO estimation, respectively.

References

[1] H. Wang and R. Song, "Low complexity ZF receiver design for multi-user GFDMA uplink systems," *IEEE Access*, vol. 6, pp. 28 661–28 667, May 2018.

[2] Z. Sharifian, M. J. Omidi, H. Saeedi-Sourck, and A. Farhang, "Linear precoding for PAPR reduction of GFDMA," *IEEE Wireless Commun. Lett.*, vol. 5, no. 5, pp. 520–523, Oct. 2016.

[3] H. Cheng, Y. Xia, Y. Huang, L. Yang, and D. P. Mandic, "Joint channel estimation and Tx/Rx I/Q imbalance compensation for GFDM systems," *IEEE Trans. Wireless Commun.*, vol. 18, no. 2, pp. 1304–1317, Feb. 2019.

[4] S. Ehsanfar, M. Matth, M. Chafii, and G. P. Fettweis, "Pilot- and CP-aided channel estimation in MIMO non-orthogonal multi-carriers," *IEEE Trans. Wireless Commun.*, vol. 18, no. 1, pp. 650–664, Jan. 2019.

[5] Y. Liu, X. Zhu, E. G. Lim, Y. Jiang, and Y. Huang, "Robust semi-blind estimation of channel and CFO for GFDM systems," in *Proc. IEEE ICC 2019*, Shanghai, China, May 2019, pp. 1–7.

[6] Z. Na, M. Zhang, M. Xiong, J. Xia, X. Liu, and W. Lu, "Pseudo-noise sequence based synchronization for generalized frequency division multiplexing in 5G communication system," *IEEE Access*, vol. 6, pp. 14 812–14 819, Mar. 2018.

[7] P. Wang and D. W. Lin, "Maximum-likelihood blind synchronization for GFDM systems," *IEEE Signal Process. Lett.*, vol. 23, no. 6, pp. 790–794, Jun. 2016.

[8] M. Huang, L. Huang, C. Guo, P. Zhang, J. Zhang, and L. Yang, "Carrier frequency offset estimation in uplink OFDMA systems: an approach relying on sparse recovery," *IEEE Trans. Veh. Tech.*, vol. 66, no. 10, pp. 9592–9597, Oct. 2017.

[9] P. Cheng, Z. Chen, F. de Hoog, and C. K. Sung, "Sparse blind carrier-frequency offset estimation for OFDMA uplink," *IEEE Trans. Comm.*, vol. 64, no. 12, pp. 5254–5265, Dec. 2016.

[10] Y. Wang and S. Phoong, "Blind CFO estimation in OFDMA uplink transmission with general carrier assignment scheme," *IEEE Comm. Lett.*, vol. 22, no. 5, pp. 1014–1017, May 2018.

[11] W. Zhang and F. Gao, "Blind frequency synchronization for multiuser OFDM uplink with large number of receive antennas," *IEEE Trans. Signal Process.*, vol. 64, no. 9, pp. 2255–2268, May 2016.

[12] H. Abdzadeh-Ziabari, W. Zhu, and M. N. S. Swamy, "Joint carrier frequency offset and doubly selective channel estimation for MIMO-OFDMA uplink with Kalman and particle filtering," *IEEE Trans. Signal Process.*, vol. 66, no. 15, pp. 4001–4012, Aug. 2018.

[13] Y. Liu, X. Zhu, E. G. Lim, Y. Jiang, Y. Huang, W. Xu, and H. Lin, "High-robustness and low-complexity joint estimation of TOAs and CFOs for multiuser SIMO OFDM systems," *IEEE Trans. Veh. Technol.*, vol. 67, no. 8, pp. 7739–7743, Aug. 2018.

[14] M. Morelli, "Timing and frequency synchronization for the uplink of an OFDMA system," *IEEE Trans. Commun.*, vol. 52, no. 2, pp. 296–306, Feb. 2004.

[15] C. Li and S. Roy, "Subspace-based blind channel estimation for OFDM by exploiting virtual carriers," *IEEE Trans. Wireless Commun.*, vol. 2, no. 1, pp. 141–150, Jan. 2003.